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Simulation of 3-D thermo-elastic fracture problems using coupled FE-EFG approach

Himanshu Pathak[a], Akhilendra Singh[a]* and Indra Vir Singh[b]

[a]*Department of Mechanical Engineering, IIT Patna, Patna 800013, India*
[b]*Department of Mechanical and Industrial Engineering, IIT Roorkee, Roorkee 247667, India*

Abstract

The present work deals with formulation and implementation of coupled FE-EFG approach to solve three dimensional crack problems in thermal loading environment. Geometrical discontinuity across crack surface is modelled by extrinsically enriched EFGM and remaining part of the domain is approximated by standard FEM. In interface/transition region, ramp function based interpolation scheme has been implemented. This coupled approach combines the advantages of both EFGM and FEM. Thermo-elastic problems are decoupled into thermal and elastic problems. At first, unknown temperature field has been obtained by solving steady-state heat conduction equation, and it is used as load input in elastic problem to get displacement and stress field. To check accuracy of proposed technique, one 3-D benchmark crack domain is solved for mechanical loading and obtained results are compared with available reference results. Further, a penny shape crack embedded in cuboid domain is simulated under three different thermo-elastic loading conditions namely thermal shock, adiabatic and isothermal loads to reveal the sturdiness and versatility of the coupled approach.

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Keywords: 3-D cracks; coupled FE-EFG; SIF; thermo-elastic cracks.

1. Introduction

Most of the critical engineering component exposed to both thermo-elastic loading during their service life. The best examples are combustion chamber of internal combustion engines, nuclear reactor component, space craft and

* Corresponding author. Tel.: +91 9631221406; fax: +91-612-227 7383.
E-mail address: akhil@iitp.ac.in

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blade casing of thermal power plant. These components failed due to combined effect of thermal and mechanical loading. Under the thermal loads, the existing voids create the stress field singularity at the tip and above the certain limitation, these voids catastrophically propagate which results the loss of property and human lives. Thus the problem requires the estimation of critical crack length and load under which component serve safely. The computation of stress intensity factor (SIF) plays an important role for the safety assessment of components. In general, all real life cracks problems are 3-D in nature. The analysis of 3-D cracked domain by analytical methods is a very tedious job. Therefore, the numerical methods are the only choice left to analyze/simulate 3-D crack problems. Although, numerical methods can simulate these problems but the accurate and efficient simulation of these cracks still remains a most challenging tasks for the fracture mechanics society. So far, the finite element method (FEM) has been largely used to solve stationary and propagating crack problems by Barsoum (1977), Gifford and Hilton (1978), Nikishkov and Atluri (1987), Rhee and Salama (1987). In finite element analyses, the geometry is normally modeled by an adequate mesh. In FEM, a crack must coincide with the edge of the finite elements i.e. a conformal mesh is needed apart from the requirement of special elements to handle crack tip asymptotic stress fields. However, the generation of good quality meshes for complex geometries and adaptive simulation, is an expensive task. The finite element simulation of cracks in 3-D becomes even more difficult and time consuming. This leads to inaccuracy in the simulation of 3-D multiple and propagating cracks. To overcome these difficulties, a number of meshfree methods were developed over past two decades. The principal attraction of the meshfree methods is their capacity to deal with moving discontinuities such as phase changes and crack propagation. A type of meshfree method, element free Galerkin method (EFGM) has been applied by Belytschko et al. (1994), Belytschko et al. (1995a) and Sukumar et al. (1997) in the fracture mechanics area. EFGM use only scattered nodal data to build the approximation. Therefore conformal meshing or remeshing is not essential for crack growth problems. One major drawback of EFGM is their high computational cost as compared to FEM. In case of 3-D crack problems, computational time becomes a critical issue, thus it is worthy to couple EFGM (meshfree) with FEM. This coupling would exploit the potential of each method while avoiding their deficiencies. The EFGM would be used near the crack surface as it can accurately model the discontinuities. The rest of domain is approximated by standard FEM. The transition between FEM and EFGM is modelled by a ramp function. The shape function obtained for interface elements include both FE and EFG shape functions coupled with ramp function. This shape function satisfies consistency condition and ensures the convergence of FE- EFG approach. 2-D crack modelling by coupled FE-EFG is already reported in the literature by Belytschko et al. (1995b), Rao and Rahman (2001), Rajesh and Rao (2010). Latter, Sukumar et al. (1997) used coupled FE-EFG approach to solve three-dimensional crack problems subjected to mechanical loading.

From the literature survey, it is clear that solution of three-dimensional crack problems under thermo-elastic loading environment have not performed so far. Therefore, in the present work, coupled FE-EFG approach has been extended to simulate three-dimensional crack problems. A new approach has been introduced to model 3-D crack surface. In this approach (Pathak et al., 2013), a crack front is divided into a number of piecewise curved parts to avoid an iterative solution. The nearest point at crack front from arbitrary (Gauss) point has been obtained for each crack segment. By using this approach, a cuboid with penny shape crack subjected to mechanical traction has been solved and obtained results are compared with solution obtained by XFEM with iterative crack modelling method. The comparative analysis performed on the basis of energy norm, convergence and involved computational time. Further the approach extended to get solution of 3-D cracked domain under different thermo-elastic loading conditions namely thermal shock, adiabatic crack and isothermal crack loads. The results are presented in form of stress plots, temperature contour and SIFs plots.

2. Numerical Formulation

Consider a three dimensional cracked domain (Ω) with boundary (Γ) is consist of Γ_c, Γ_T, Γ_t and Γ_u is shown in Fig 1. The linear heat conduction equation for thermal field solution can be written as:

$$-\nabla \mathbf{q} + \mathrm{Q} = 0, \ \mathbf{q} = -k\nabla T \tag{1}$$

And equilibrium equation and boundary conditions for linear elastic crack problem may be described as:

$$\nabla:\boldsymbol{\sigma}+\mathbf{b}=0 \text{ in } \Omega,\ \boldsymbol{\sigma}=\mathbf{C}:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}_T),\ \boldsymbol{\sigma}.\mathbf{n}=\bar{\mathbf{t}} \quad \text{on } \Gamma_t,\ \boldsymbol{\sigma}.\mathbf{n}=0 \quad \text{on } \Gamma_{c^+}/\Gamma_{c^-} \tag{2}$$

Here, thermo-elastic strain tensor is defined as $\boldsymbol{\varepsilon}=\nabla_s\mathbf{u}$ and $\boldsymbol{\varepsilon}_\mathbf{T}=\alpha(T-T_{ref})\mathbf{I}$ (3)

In the above governing equation, $\mathbf{q}$ is heat flux vector, Q represents the heat source, k is the thermal conductivity of materials, $\mathbf{n}$ is unit outward normal on the boundaries, $\mathbf{u}$ is displacement field, $\boldsymbol{\sigma}$ is Cauchy stress tensor, $\mathbf{b}$ is the body forces per unit volume, $\mathbf{C}$ is isotropic fourth order tensor, ∇_s is the symmetric gradient operator, T is the temperature field within the domain, $\boldsymbol{\varepsilon}$ is the strain vector, $\boldsymbol{\varepsilon}_\mathbf{T}$ is the thermal strain vector with respect to reference temperature T_{ref}, α is the thermal expansion coefficient and $\mathbf{I}$ represents the second order identity tensor. In order to model 3D cracked body with discontinuity in displacement and temperature field, extrinsic enriched approximation within EFGM framework has been given by Duflot (2008) as:

$$T^h(\mathbf{x})=\underbrace{\sum_{I\in \mathrm{N}}\varphi_I(\mathbf{x})\,T_I}_{classical}+\underbrace{\sum_{J\in \mathrm{N}^c}\varphi_J(\mathbf{x})H(\mathbf{x})\,a_J}_{Heaviside\,enrichment}+\underbrace{\sum_{K\in \mathrm{N}^f}\varphi_K(\mathbf{x})\sum_{\alpha=1}^{4}B_\alpha(\mathbf{x})b_K^\alpha}_{branch\,enrichment} \quad \langle adiabatic\ \ crack\rangle \tag{4}$$

$$T^h(\mathbf{x})=\underbrace{\sum_{I\in \mathrm{N}}\varphi_I(\mathbf{x})\,T_I}_{classical}+\sum_{K\in \mathrm{N}^f}\varphi_K(\mathbf{x})\underbrace{\sqrt{R}\cos\frac{\theta}{2}b_K}_{heat\,flux\,discontinuity} \quad \langle isothermal\ \ crack\rangle \tag{5}$$

$$\mathbf{u}^h(\mathbf{x})=\underbrace{\sum_{I\in \mathrm{N}}\varphi_I(\mathbf{x})\,\mathbf{u}_I}_{classical}+\underbrace{\sum_{J\in \mathrm{N}^c}\varphi_J(\mathbf{x})H(\mathbf{x})\,c_J}_{Heaviside\,enrichment}+\underbrace{\sum_{K\in \mathrm{N}^f}\varphi_K(\mathbf{x})\sum_{\alpha=1}^{4}B_\alpha(\mathbf{x})d_K^\alpha}_{branch\,enrichment} \tag{6}$$

where, $\varphi(\mathbf{x})$ are EFGM shape functions, *a*, *b*, *c* and *d* are the additional unknowns associated with temperature and displacement based enriched approximation. Temperature field approximation is not required for thermal shock problems; thermal stress has been evaluated by constant temperature gradient (∇T). The set N^c includes the nodes whose support domain contains evaluation point **x** and it is cut by the crack surface called as split nodes, whereas set N^f are nodes whose support domain contains evaluation point **x** and the crack front. It is known as tip nodes. The Heaviside function $H(\mathbf{x})$ is given by Mohammadi (2008) as:

$$H(\mathbf{x})=\begin{cases}+1 & \text{if } (\mathbf{x}-\mathbf{x}^*)\cdot n\geq 0\\ -1 & \text{otherwise}\end{cases} \tag{7}$$

where, $\mathbf{x}^*$ be the closest point of projection to **x** (evaluation point) on the crack surface, and *n* be the normal to the crack plane.

The branch functions B_α is given by Fleming et al. (1997) and Mohammadi (2998) as:

$$B_\alpha=[\mathrm{B}_1,\mathrm{B}_2,\mathrm{B}_3,\mathrm{B}_4]=\left[\sqrt{R}\sin\frac{\theta}{2},\sqrt{R}\cos\frac{\theta}{2},\sqrt{R}\sin\frac{\theta}{2}\sin\theta,\sqrt{R}\cos\frac{\theta}{2}\sin\theta\right] \tag{8}$$

where, R and θ are polar coordinates of evaluation point in the local crack front coordinate.

2.1. Coupled FE-EFG

To maintain consistency in transition/interface region (as shown in Fig. 2), primary variable is approximated by ramp function based shape functions. Figure 2 show the domain Ω splits in two parts namely Ω^{FE} and Ω^{EFG}. These two regions are interfaced using transition element in the interface domain Ω^{INT}. Approximation of displacement field using ramp function is given by Belytschko et al. (1995b) as:

$$u_I^h(\mathrm{x}) = u_I^{\mathbf{FE}}(\mathrm{x}) + Rm(\mathrm{x})[u_I^{\mathbf{EFG}}(\mathrm{x}) - u_I^{\mathbf{FE}}(\mathrm{x})] = [1 - Rm(\mathrm{x})]u_I^{\mathbf{FE}}(\mathrm{x}) + Rm(\mathrm{x})u_I^{\mathbf{EFG}}(\mathrm{x}) \tag{9}$$

where, $\mathrm{x} \in \Omega^{INT}$, $u_I^{\mathbf{FE}}$ and u_I^{EFG} are the FEM and EFGM approximations and $Rm(\mathbf{x})$ is the ramp function defined as the sum of the FEM shape functions associated with interface element nodes on EFGM boundary.

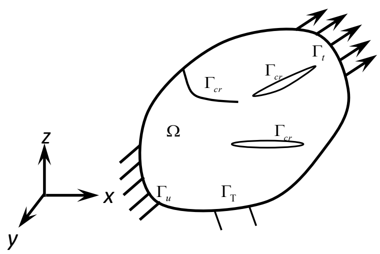

Fig. 1. Domain with a discontinuity.

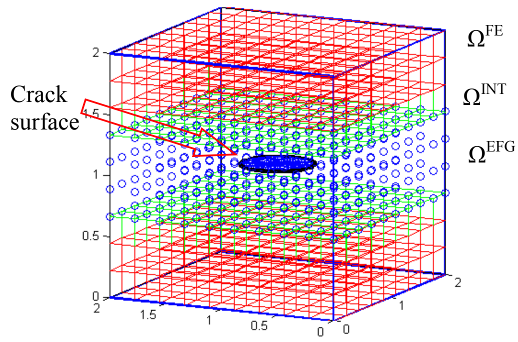

Fig. 2. Coupled discretization of domain.

$$Rm(\mathbf{x}) = \sum_{\substack{J \\ \mathbf{x}_J \in \Gamma_{\mathrm{EFG}}}} N_J(\mathbf{x}) \tag{10}$$

Substituting displacement approximations in Eq. (9) to obtain shape function in transition element, is given by

$$\begin{aligned} u_I^h(\mathbf{x}) &= [1 - Rm(\mathbf{x})]\sum_{J=1}^{n_{FE}} N_J(\mathbf{x})u_{IJ} + Rm(\mathbf{x})\sum_{J=1}^{n_{EFG}} \varphi_J(\mathbf{x})u_{IJ}\,, \quad \mathbf{x} \in \Omega^{INT} \\ &\equiv \sum_{J=1}^{n_{INT}} \chi_J(\mathbf{x})u_{IJ}\,, \qquad \mathbf{x} \in \Omega^{INT} \end{aligned} \tag{11}$$

Therefore, coupled shape function for transition region can be written as:

$$\chi_J(\mathbf{x}) = [1 - Rm(\mathbf{x})]N_J(\mathbf{x}) + Rm(\mathbf{x})\varphi_J(\mathbf{x}) \tag{12}$$

In general, shape function for whole computational domain are summarised as:

$$\tilde{N}_I(\mathbf{x}) = \begin{cases} N_I(\mathbf{x}), & \mathbf{x} \in \Omega^{\mathrm{FE}} \\ \varphi_I(\mathbf{x}), & \mathbf{x} \in \Omega^{\mathrm{EFG}} \\ \chi_I(\mathbf{x}), & \mathbf{x} \in \Omega^{\mathrm{INT}} \end{cases} \tag{13}$$

Here, $N_I(\mathbf{x})$ is FEM shape function, $\varphi_I(\mathbf{x})$ is standard MLS based EFGM shape function and $\chi_I(\mathbf{x})$ is ramp function based coupled FE-EFG shape function.

2.2. SIF Evaluation

In the present work, domain based interaction integral approach is used to extract the numerical SIFs. A crack front contour integral is expressed in terms of volume integral over the domain surrounding the crack front. Thermal interaction integral is described by Sills and Dolev (2004) in the following way:

$$M^{(1,2)} = \int_{\Omega}\left[\sigma_{ij}^{(1)}\frac{\partial u_i^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)}\frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2)}\delta_{ij}\right]\frac{\partial q}{\partial x_j}dA + \alpha\int_A \frac{\partial T}{\partial x_1}\sigma_{kk}^{(2)}q\,d\Omega \tag{14}$$

where, q is a scalar weight function, $\sigma_{ij}^{(1)}$ and $\varepsilon_{ij}^{(1)}$, are the actual Cauchy stress and engineering strain, while $\sigma_{ij}^{(2)}$ and $\varepsilon_{ij}^{(2)}$ are the auxiliary Cauchy stress and engineering strain. $W^{(1,2)}$ is strain energy density at actual and auxiliary states.

3. Numerical Examples

Coupled EFGM-FEM approach has been used for 3-D thermo-elastic crack modelling. The whole domain sub-divided into two parts. EFGM is implemented near to crack surface; as it can accurately capable of solve crack problems based on nodal approximation only. The rest of domain is simulated by standard FEM. The transition/interface region is modelled by ramp function based interpolation scheme. FEM approximation has been performed using eight nodded brick element. In MLS based approximation, quadratic spline weight function has been used. Optimum scaling factor has been selected to define domain of influence so that non-singular solution can be achieved and local crack effect can be captured. In this work, scaling factor to define domain of influence for standard EFGM approximation, Heaviside enriched approximation and branch function enriched approximation are taken as *1.25*, *1.00* and *1.00* respectively. Standard Gauss quadrature has been used for numerical integration. Fourth order Gauss point considered for EFGM and interface computational domain whereas second order Gauss quadrature is taken for FEM based simulation region. All numerical simulation performed on Acer System Core i5-2500 CPU @ 3.30GHz (4GB RAM) machine with 64 bit Windows 7 Professional operating system. An in-house MATLAB code has been developed to obtain all presented numerical results. Following material properties are used in presented simulation works:

Table 1. Material Property for Simulation.

S. No.	Parameter	Numerical value
1.	Elastic modulus (E)	$200 GPa$
2.	Poisson ratio (ν)	0.3
3.	Thermal expansion coefficient (α)	11.7×10^{-6} /ºC
4.	Thermal conductivity (K)	250 W/mK

3.1. Mechanical Load

To check accuracy and efficiency of coupled method, a penny shape crack problem subjected to mechanical traction load solved by EFGM, XFEM and coupled method as shown in Fig 3.

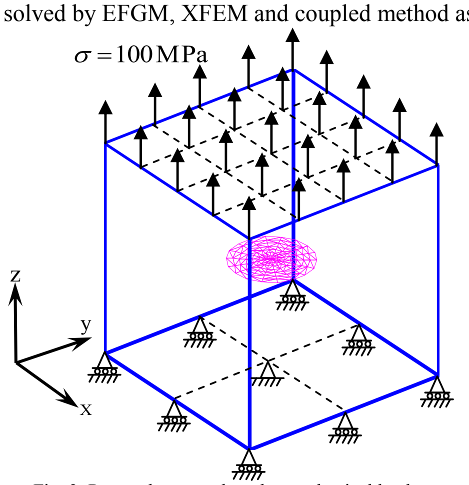

Fig. 3. Penny shape crack under mechanical load.

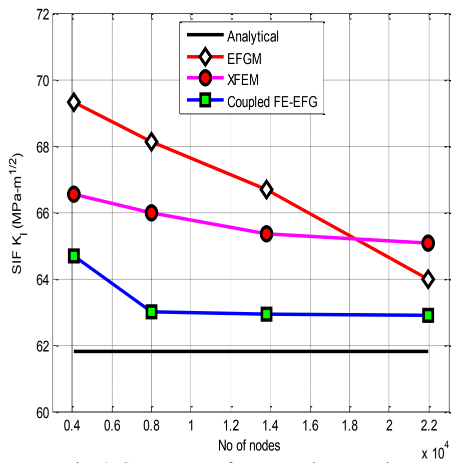

Fig. 4. Convergence for penny shape crack.

A cuboid $(2m \times 2m \times 2m)$ with a penny shape crack of radius 0.3 m lying at cuboid centre has been considered for the analysisEfficiency of these methods is presented in terms of convergence rate and error in energy norm. Fig 4 and Fig 5 show the coupled method is more accurate than other methods to solve crack domain problem. Fig 6 represents the computational time comparison solved by XFEM with existing iterative approach and introduced segmentation approach using EFGM, XFEM and coupled FE-EFG. Based on these comparative analyses, it can be stated that crack surface segmentation approach is computationally efficient to model 3-D crack geometry.

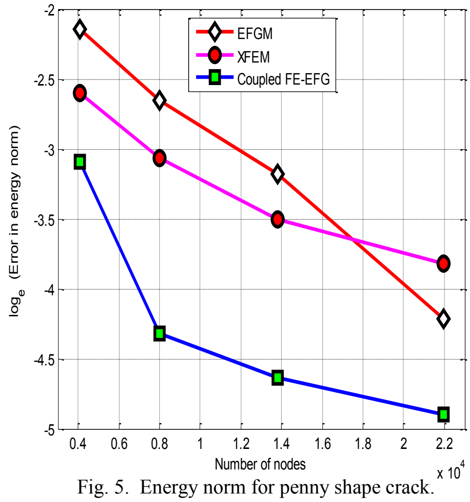

Fig. 5. Energy norm for penny shape crack.

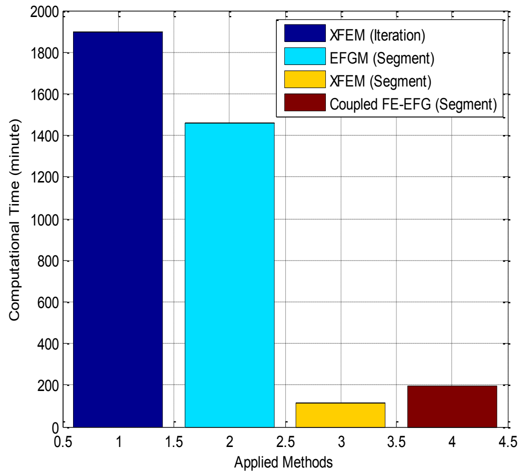

Fig. 6. Computational time comparison for penny shape crack.

3.2. Thermal Shock Load

In this sub-section, a penny shape crack embedded in cuboid has been analysed under thermal shock load of $(\Delta T) = -20^o C$. Top and bottom surfaces of cuboid are constrained in *x*, *y* and *z* directions. A problem domain along with applied boundary conditions is shown in Fig 7.

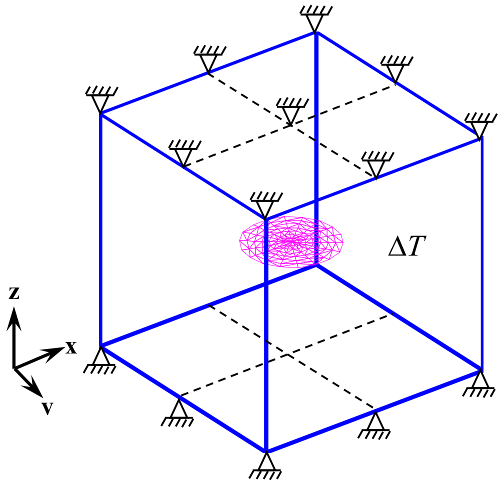

Fig. 7. Penny shape crack under thermal shock load.

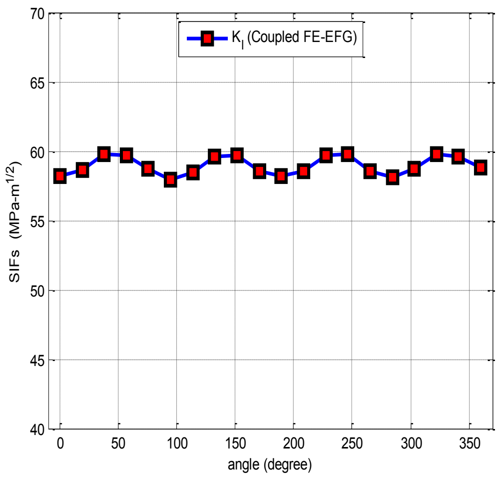

Fig. 8. SIFs variation along crack front.

The uniform thermal shock load under the imposed boundary conditions induces mode-I loading on the crack. The values of numerical SIF are plotted in Fig 8. Stress field contour of σ_{zz} obtained at the crack surface and presented in Fig 9. From this stress contour plot, a stress field singularity can be clearly visualized in presented magnified stress plot of crack surface.

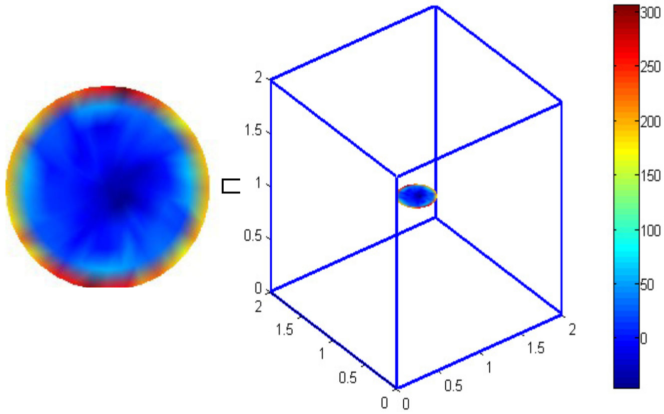

Fig. 9. Stress field contour σ_{zz} under thermal shock load.

3.3. Adiabatic Crack Load

In adiabatic crack case, crack surfaces are assumed as thermally insulated, and a specific temperature is imposed at outer boundary of cracked geometry. For an adiabatic crack, both displacement and temperature fields become discontinuous across the crack surface, and heat flux is found singular at the crack tip. A cuboid with penny crack surface has been numerically solved under adiabatic crack loading condition. The radius of penny is taken as $r = 0.3m$. Problem domain along with applied boundary conditions is shown in Fig 10.

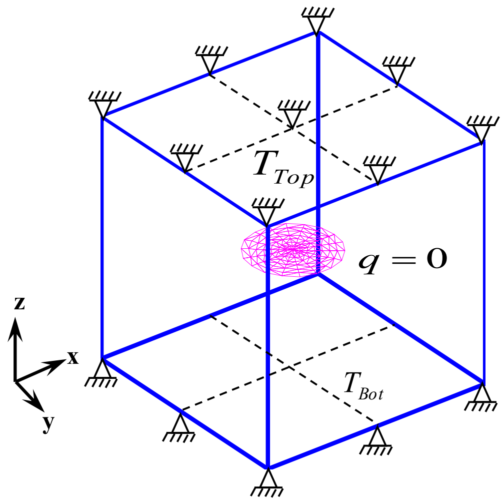

Fig. 10. Penny shape crack under adiabatic thermal load.

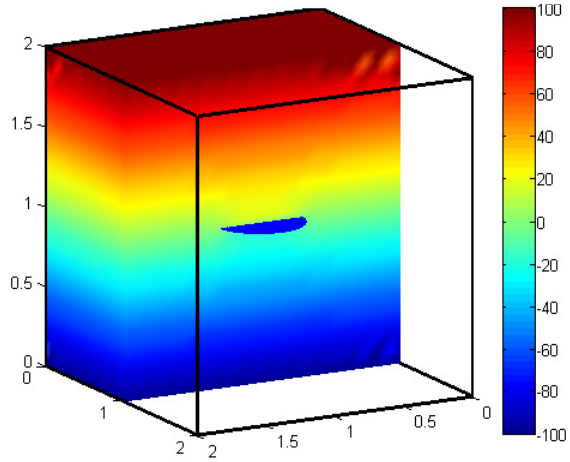

Fig. 11. Half section temperature contour for a penny shape crack domain.

Specified temperatures i.e. ($T_{Top} = 100^o C$, $T_{Bot} = -100^o C$) are imposed on the top and bottom faces of cuboid domain and crack surface assumed as thermally insulated. To visualize discontinuity in temperature field across the crack surface, half sectional cuboid temperature contours are presented in Fig 11. As expected, discontinuity in temperature contour across crack surface has been captured. The numerically obtained mode-I and mode-II SIFs are presented in Fig 12. SIF plot shows that mode-II SIFs are more dominating.

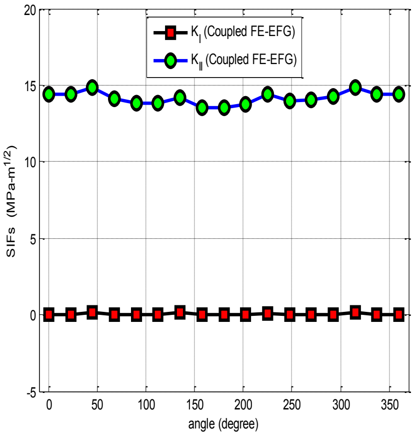

Fig. 12. SIFs variation along crack front.

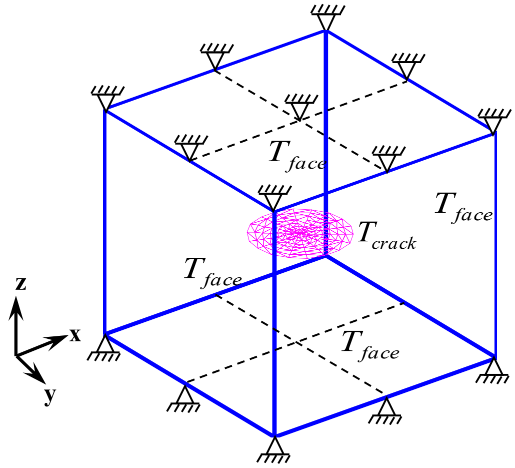

Fig. 13. Penny shape cracked domain under isothermal crack load.

3.4. Isothermal Crack Load

In case of an isothermal crack, the essential boundary condition i.e. temperature is imposed at the crack surface ($T = \overline{T}$ on Γ_c) as well as on all outer surfaces of cuboid. As a result, uniform radial heat flux maintained between crack surface and outer boundary of problem domain. As crack surface kept at specified temperature therefore this kind of boundary condition named as isothermal crack problems. A penny shape cracked domain with isothermal crack loading condition has been considered for the simulation. Problem domain along with applied boundary conditions is shown in Fig 13. The temperatures are specified ($T_{face} = 100^o C$ and $T_{crack} = 0^o C$) at outer cuboid and crack surface. Sectional temperature contour of half body with crack surface has been presented in Fig 14. As expected radial variation in temperature contour has been obtained. The numerically obtained mode-I, mode-II and mode-III SIFs are presented in Fig 15. SIF plot shows that mode-I SIFs are dominating in nature and almost constant throughout crack front but mode-II and mode-III SIFs are near zero along crack front.

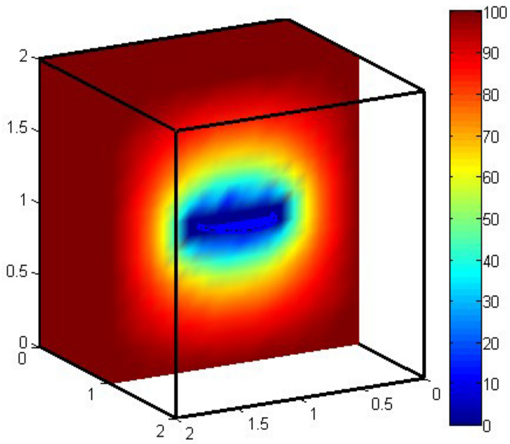

Fig. 14. Half section temperature contour for a penny shape crack domain.

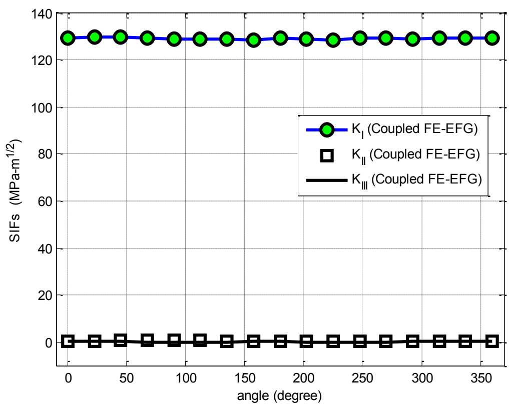

Fig. 15. SIFs variation along crack front.

4. Conclusions

In this paper, 3-D thermo-elastic crack simulations have been performed by coupled FE-EFG approach. Geometrical discontinuity across crack surface is modelled by extrinsically enriched EFGM while remaining part of the domain is approximated by standard FEM. In interface/transition region, ramp function based interpolation scheme has been implemented. This coupled approach combines the advantages of both EFGM and FEM. A generalized MATLAB code has been developed to simulate various 3-D linear elastic crack problems under mechanical and thermo-elastic loads. The modified thermal interaction integral approach has been used to extract mixed mode SIFs. The results show that coupled FE-EFG approach is quite accurate and efficient to model 3-D cracked geometries. It has been seen that thermal shock and isothermal loading creates mode-I failure effect whereas adiabatic thermal loading induce mode-II dominating failure effect. This work can be further extended to solve 3-D crack growth problems under thermo-mechanical loading in nonlinear materials.

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